Fast and Guaranteed Tensor Decomposition via Sketching

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December 2, 2015

Accelerated tensor low-rank decomposition

Symmetric tensor CP decomposition

For symmetric tensor $T \in \mathbb{R}^{n \times n \times n}$, find $\{(\lambda_i, u_i)\}_{i=1}^k$ to minimize

$$\|T - \textstyle\sum_{i=1}^k \lambda_i u_i^{\otimes 3}\|_F^2.$$

- Wide application in data mining and latent variable models.
- **Tensor power iteration**: $u^{(t+1)} = T(I, u^{(t)}, u^{(t)}) / \|T(I, u^{(t)}, u^{(t)})\|_2$.
- Accelerated tensor power iteration via sketching:
  - TensorSketch: $s(T) \in \mathbb{R}^b$, for $n < b \ll n^3$.

$$\begin{aligned} [T(I,u,u)]_i &\approx \langle s(T), \quad s(u \otimes u \otimes e_i)\rangle \\ &= \langle \mathcal{F}(s(T)), \quad \mathcal{F}(s(u)). * \mathcal{F}(s(u)). * \mathcal{F}(s(e_i))\rangle \\ &= \langle \mathcal{F}^{-1}(\mathcal{F}(s(T)). * \overline{\mathcal{F}(s(u))}. * \overline{\mathcal{F}(s(u))}), \quad s(e_i)\rangle. \end{aligned}$$

  - Time complexity: $O(n^3) \rightarrow O(n + b \log b)$.

Efficient spectral method for topic modeling

Topic modeling

V: vocabulary size; k: number of topics. Recover topic distributions $\mu_1, \cdots, \mu_k \in \mathbb{R}^V$ from N unlabeled documents.

Figure 1: Negative log-likelihood and running time (min) on Wikipedia dataset.

k		like.	time	$\log_2 b$	iters	k	like.	time	$\log_2 b$	iters
200	Spectral	7.49	**34**	12	-	300	7.39	**56**	13	-
	Gibbs	6.85	561	-	30		6.38	818	-	30
	Hybrid	**6.77**	144	12	5		**6.31**	352	13	10